My Explorations in the Collatz-Forest and Some of My Findings

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SOME ICE-BREAKER PLEASANTRIES

we may not be in an integral domain;

but, i can assure you, there are

no irrationals; no radicals; no trans; no imaginaries; nothing complex;

not even any negatives;

only positives, that too, with some modular ideals.

HISTORICAL BACKGROUND

Lothar Collatz (1910-1990); **Bryan Thwaites (1923-);** **Stanisław Ulam (1909-1984);**
Helmut Hasse (1898-1979); **Syracuse University, NY;** **Shizuo Kakutani (1911-2004);**

HISTORICAL BAGGAGE

Jeffrey C. Lagarias refers to the statement by Paul Erdos,
one of the most revered Mathematician and Number Theory Expert, who said that -
"Mathematics is not yet ready for such problems" – "Hopeless, Absolutely Hopeless" –
about the Collatz Conjecture.

Richard K. Guy has been somewhat pessimistic in advising researchers -
"Don't Try to Solve These Problems".

Jeffrey C. Lagarias gives an elaborate overview of the Collatz Problem,
in his book "The Ultimate Challenge: The 3x+1 Problem";
and also an extensive two-part annotated bibliography.
"It is hard to resist exploring its structure. We should not exclude it from the mathematical universe just because we are unhappy with its difficulty. It is a fascinating and addictive problem".

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Collatz Function **Collatz (3x+1) Sequence** **Collatz Conjecture**

Start with any given positive integer x. If x is even then, divide by 2 {x ⇐ x/2}; else(if x is odd) then, multiply by 3 and add 1 {x ⇐ (3x+1)} so as to get a resultant that is even. Now continue the process.

My way of defining the Collatz Function:

if (x≡0(mod2)) then S(x) ≔ [x]D ≔ (x/2);
elseif (x≡1(mod2)) then S(x) ≔ [x]U ≔ (3x+1);

The sequence thus generated, starting from any given positive integer, is the Collatz Sequence corresponding to that given positive integer.

The Collatz-Thwaites-Ulam-Hasse-Syracuse-Kakutani (CTUHSK) Sequence

The Collatz Conjecture states that the Collatz Sequence will eventually converge to the trivial-cycle; that is, {(1⇐2⇐4)}; starting from any positive integer.

Collatz Sequence starting from 27

27, 82, **41**, 124, 62, **31**, 94, **47**, 142, **71**, 214, **107**, 322, **161**, 484, 242, **121**, 364, 182, **91**, 274, **137**, 412, 206, **103**, 310, **155**, 466, **233**, 700, 350, **175**, 526, **263**, 790, **395**, 1186, **593**, 1780, 890, **445**, 1336, 668, 334, **167**, 502, **251**, 754, **377**, 1132, 566, **283**, 850, **425**, 1276, 638, **319**, 958, **479**, 1438, **719**, 2158, **1079**, 3238, **1619**, 4858, **2429**, 7288, 3644, 1822, **911**, 2734, **1367**, 4102, **2051**, 6154, **3077**, 9232, 4616, 2308, 1154, **577**, 1732, 866, **433**, 1300, 650, **325**, 976, 488, 244, 122, **61**, 184, 92, 46, **23**, 70, **35**, 106, **53**, **160, 80, 40, 20, 10, 5**, 16, 8, 4, 2, **1**

27, 82, **41**, 124, **31**, 94, **47**, 142, **71**, 214, **107**, 322, **161**, 484, **121**, 364, **91**, 274, **137**, 412, **103**, 310, **155**, 466, **233**, 700, **175**, 526, **263**, 790, **395**, 1186, **593**, 1780, **445**, 1336, **167**, 502, **251**, 754, **377**, 1132, **283**, 850, **425**, 1276, **319**, 958, **479**, 1438, **719**, 2158, **1079**, 3238, **1619**, 4858, **2429**, 7288, **911**, 2734, **1367**, 4102, **2051**, 6154, **3077**, **9232**, **577**, 1732, 866, **433**, 1300, 650, **325**, 976, 488, 244, 122, **61**, 184, 92, 46,
23, 70, **35**, 106, **53**, **160, 5**, 16, **1**

27; 41; 31; 47; 71; 107; 161; 121; 91; 137; 103; 155; 233;
175; 263; 395; 593; 445; 167; 251; 377; 283; 425;
319; 479; 719; 1079; 1619; 2429; 911; 1367; 2051; 3077; 577; 433; 325; 61; 23;
35; 53; 5; 1

CTUHSK-Generative-Parameter-Tuple for 27 : Reverse-Collatz-Halvings-Tuple
(4, 5, 1, 1, 3, 4, 2, 2, 4, 1, 1, 1, 3, 1, 1, 1, 1, 1, 2, 1, 2, 1, 1, 3, 2, 1, 1, 1, 2, 1, 1, 2, 1, 2, 2, 1, 1, 1, 1, 2, 1, 0)

Compact Collatz (even-to-odd, odd-to-even) Function

if $x\equiv 0(\mathrm{mod}2)$ then, $T(x) \coloneqq [x]D^{\#} \coloneqq [x]D^{*} \coloneqq [x]D^{p} = (x/2^{p})$; for some valid $p\geq 1$;
so as to get a resultant that is odd, $x\not\equiv 0(\mathrm{mod}2)$;
elseIf $x\not\equiv 0(\mathrm{mod}2)$ then, $T(x) \coloneqq [x]U^{\#} \coloneqq [x]U \coloneqq (3x+1)$; which is even, $x\equiv 0(\mathrm{mod}2)$.

The compact Collatz sequence is an alternating sequence of odd and even numbers.

Condensed Compact (odd-to-odd) CTUHSK Function

Start with any positive odd integer $x>1$; $x\not\equiv 0(\mathrm{mod}2)$; apply the operator $(U^{\#}D^{\#})$;
$x \Leftarrow [x](U^{\#}D^{\#})$; so that the resultant is odd, $x\not\equiv 0(\mathrm{mod}2)$; now continue the process.

Condensed Compact (odd-to-odd) Inverse CTUHSK Function

if $\{(y\equiv 1(\mathrm{mod}6))\vee((y\equiv 5(\mathrm{mod}6))\}$ then $\{x\} \coloneqq \{P(y)\} \coloneqq (y.2^{k} - 1)/3\}$; to get a positive integer output x.

input $y \coloneqq 6b+x_6$; with $0\leq b$; $b_3 \coloneqq b(\mathrm{mod}3)\in\{1,2,0\}$; $y_3 \coloneqq y(\mathrm{mod}3)\in\{1,2\}$ and $y_6 \coloneqq y(\mathrm{mod}6)\in\{1,5\}$;
binary-exponent-value $k \coloneqq 2j - (y_6 - y_3)/3$; for the j^{th} jump; $1\leq j$; $j_3 \coloneqq j(\mathrm{mod}3)\in\{1,2,0\}$;
output $x \coloneqq 6a+x_6$; with $0\leq a$; $a_3 \coloneqq a(\mathrm{mod}3)\in\{1,2,0\}$; $x_3 \coloneqq x(\mathrm{mod}3)\in\{1,2,0\}$ and $x_6 \coloneqq x(\mathrm{mod}6)\in\{1,5,3\}$;

Modular Periodicity

$[b]_3,[y]_6$ =	(0,1)	(0,5)	(1,1)	(1,5)	(2,1)	(2,5)	= $[b]_3,[y]_6$
j=9; w=18 j=9; v=17	**(1,3)**87381	218453**(0,5)**	**(1,5)**611669	480597**(2,3)**	**(2,1)**1135957	742741**(1,1)**	j=9; w=18 j=9; v=17
j=8; w=16 j=8; v=15	**(1,5)**21845	54613**(0,1)**	**(1,1)**152917	120149**(2,5)**	**(0,3)**283989	185685**(2,3)**	j=8; w=16 j=8; v=15
j=7; w=14 j=7; v=13	**(1,1)**5461	13653**(1,3)**	**(2,3)**38229	30037**(2,1)**	**(0,5)**70997	46421**(2,5)**	j=7; w=14 j=7; v=13
j=6; w=12 j=6; v=11	**(2,3)**1365	3413**(1,5)**	**(2,5)**9557	1251**(0,3)**	**(0,1)**17749	11605**(2,1)**	j=6; w=12 j=6; v=11
j=5; w=10 j=5; v= 9	**(2,5)**341	853**(1,1)**	**(2,1)**2389	1877**(0,5)**	**(1,3)**4437	2901**(0,3)**	j=5; w=10 j=5; v= 9
j=4; w= 8 j=4; v= 7	**(2,1)**85	213**(2,3)**	**(0,3)**597	469**(0,1)**	**(1,5)**1109	725**(0,5)**	j=4; w= 8 j=4; v= 7
j=3; w= 6 j=3; v= 5	**(0,3)**21	53**(2,5)**	**(0,5)**149	117**(1,3)**	**(1,1)**277	181**(0,1)**	j=3; w= 6 j=3; v= 5
j=2; w= 4 j=2; v= 3	**(0,5)**5	13**(2,1)**	**(0,1)**37	29**(1,5)**	**(2,3)**69	45**(1,3)**	j=2; w= 4 j=2; v= 3
j=1; w= 2 j=1; v= 1	**(0,1)**1	3**(0,3)**	**(1,3)**9	7**(1,1)**	**(2,5)**17	11**(1,5)**	j=1; w= 2 j=1; v= 1
$[b]_3,[y]_6$ =	(0,1)	(0,5)	(1,1)	(1,5)	(2,1)	(2,5)	= $[b]_3,[y]_6$

$[b]_3,[y]_6$ =	(0,1)	(0,5)	(1,1)	(1,5)	(2,1)	(2,5)	(0,1)	(0,5)	(1,1)	(1,5)	(2,1)	(2,5)	(0,1)	(0,5)	(1,1)	(1,5)	(2,1)	(2,5)	= $[b]_3,[y]_6$
j=1; w= 2	(0,1)	3	(1,3)	7	(2,5)	11	(1,1)	15	(2,3)	19	(0,5)	23	(2,1)	27	(0,3)	31	(1,5)	35	j=1; w= 2
j=1; v= 1	1	(0,3)	9	(1,1)	17	(1,5)	25	(2,3)	33	(0,1)	41	(0,5)	49	(1,3)	57	(2,1)	65	(2,5)	j=1; v= 1
$[b]_3,[y]_6$ =	(0,1)	(0,5)	(1,1)	(1,5)	(2,1)	(2,5)	(0,1)	(0,5)	(1,1)	(1,5)	(2,1)	(2,5)	(0,1)	(0,5)	(1,1)	(1,5)	(2,1)	(2,5)	= $[b]_3,[y]_6$

SOME SIMPLER CREATURES

Halemane's (1.x+1)/2 System (even-to-odd, odd-to-even) Form

Start with any given positive integer x;
If $x \equiv 0 \pmod 2$ then, divide by 2 repeatedly, until the resultant $x \equiv 1 \pmod 2$;
ElseIf $x \equiv 1 \pmod 2$ then, add 1 to get a resultant $x \equiv 0 \pmod 2$; Now continue the process.

if $x \equiv 0 \pmod 2$ then, $T(x) := [x]D^{\#} := [x]D^{*} := [x]D^{p} = (x/2^{p})$;
elseIf $x \equiv 1 \pmod 2$ then, $T(x) := (1.x+1)$; so that the resultant is even.

Inverse of (1.x+1)/2 System - Condensed Compact (odd-to-odd) Form

if $y \equiv 1 \pmod 2$ then $\{P(y)\} := \{P_1(y, u) := (y.2^{u} - 1)\}$; $1 \leq u$.

Halemane's (2.x±1)/3 System

Start with any given positive integer x;
If $x \equiv 0 \pmod 3$ then, divide by 3 repeatedly until the resultant $x \not\equiv 0 \pmod 3$;
ElseIf $x \equiv 1 \pmod 3$ then, multiply by 2 and add 1 to get a resultant $x \equiv 0 \pmod 3$;
ElseIf $x \equiv 2 \pmod 3$ then, multiply by 2 and subtract 1 to get a resultant $x \equiv 0 \pmod 3$; Now continue the process.

Inverse of (2.x±1)/3 System

if $\{(y \equiv 1 \pmod 6) \vee ((y \equiv 5 \pmod 6))\}$ then $\{x\} := \{P(y)\} := (y.3^{u} \pm 1)/2\}$; with $1 \leq u$.

Halemane's (3.x±1)/4 System

Start with any given positive integer x>1; if (x=1) STOP;
elseIf x≡0(mod4) then, divide by 4 repeatedly until the resultant x≢0(mod4);
ElseIf x≡2(mod4) then, divide by 2 to get a resultant x≢2(mod4);
ElseIf {x≡1(mod4)} then, multiply by 3 and add 1 to get a resultant {x≡0(mod4)};
ElseIf {x≡3(mod4)} then, multiply by 3 and subtract 1 to get a resultant {x≡0(mod4)};
Now continue the process.

Halemane's (3.x±1)/4 Function

if (x=1) STOP;
elseIf x≡0(mod4) then, S(x) ≔ (x/4);
elseIf x≡2(mod4) then, S(x) ≔ (x/2);
elseIf {x≡1(mod4)}} then, S(x) ≔ (3.x+1);
elseIf {x≡3(mod4)}} then, S(x) ≔ (3.x-1);

	$[0]_4$	$[1]_4$	$[2]_4$	$[3]_4$
$[0]_4$	0	1	1	1
$[1]_4$	1	0	0	0
$[2]_4$	0	1	0	1
$[3]_4$	1	0	0	0

condensed compact (odd-to-odd) inverse of Halemane's (3.x±1)/4 function

if {(y≡1(mod6))∨((y≡5(mod6))} then {x} ≔ {P(y)} ≔ $(4.y.2^u \pm 1)/3$}; with 0≤u.

Halemane's (3.x±1)/4 System -vis-à-vis- Collatz (3x+1)/2 System

	1	5	7	11	13	17	19	23	25	29	31	35	37	41	43	47	49	53	
19	174763 (0,1)	873813 (0,3)	1223339	1922389	2271915	2970965	3320491	4019541	4369067	5068117	5417643	6116693	6466219	7165269	7514795	8213845	8563371	9262421	19
18	87381 (1,3)	436907 (1,5)	611669	961195	1135957	1485483	1660245	2009771	2184533	2534059	2708821	3058347	3233109	3582635	3757397	4106923	4281685	4631211	18
17	43691 (0,5)	218453 (0,5)	305835	480597	567979	742741	830123	1004885	1092267	1267029	1354411	1529173	1616555	1791317	1878699	2053461	2140843	2315605	17
16	21845 (1,5)	109227 (0,3)	152917	240299	283989	371371	415061	502443	546133	633515	677205	764587	808277	895659	939349	1026731	1070421	1157803	16
15	10923 (2,3)	54613 (0,1)	76459	120149	141995	185685	207531	251221	273067	316757	338603	382293	404139	447829	469675	513365	535211	578901	15
14	5461 (1,1)	27307 (0,1)	38229	60075	70997	92843	103765	125611	136533	158379	169301	191147	202069	223915	234837	256683	267605	289451	14
13	2731 (2,1)	13653 (1,3)	19115	30037	35499	46421	51883	62805	68267	79189	84651	95573	101035	111957	117419	128341	133803	144725	13
12	1365 (2,3)	6827 (0,5)	9557	15019	17749	23211	25941	31403	34133	39595	42325	47787	50517	55979	58709	64171	66901	72363	12
11	683 (2,5)	3413 (1,5)	4779	7509	8875	11605	12971	15701	17067	19797	21163	23893	25259	27989	29355	32085	33451	36181	11
10	341 (2,5)	1707 (2,3)	2389	3755	4437	5803	6485	7851	8533	9899	10581	11947	12629	13995	14677	16043	16725	18091	10
9	171 (1,3)	853 (1,1)	1195	1877	2219	2901	3243	3925	4267	4949	5291	5973	6315	6997	7339	8021	8363	9045	9
8	85 (2,1)	427 (2,1)	597	939	1109	1451	1621	1963	2133	2475	2645	2987	3157	3499	3669	4011	4181	4523	8
7	43 (1,1)	213 (2,3)	299	469	555	725	811	981	1067	1237	1323	1493	1579	1749	1835	2005	2091	2261	7
6	21 (0,3)	107 (2,5)	149	235	277	363	405	491	533	619	661	747	789	875	917	1003	1045	1131	6
5	11 (1,5)	53 (2,5)	75	117	139	181	203	245	267	309	331	373	395	437	459	501	523	565	5
4	5 (0,5)	27 (1,3)	37	59	69	91	101	123	133	155	165	187	197	219	229	251	261	283	4
3	3 (0,3)	13 (2,1)	19	29	35	45	51	61	67	77	83	93	99	109	115	125	131	141	3
2	1 (0,1)	7 (1,1)	9 (1,3)	15 (2,3)	17 (2,5)	23 (0,5)	25 (1,1)	31 (2,1)	33 (2,3)	39 (0,3)	41 (0,5)	47 (1,5)	49 (2,1)	55 (0,1)	57 (0,3)	63 (1,3)	65 (1,5)	71 (2,5)	2
1		3		7		11		15		19		23		27		31		35	1
	1	5	7	11	13	17	19	23	25	29	31	35	37	41	43	47	49	53	

Collatz (3x+1)/2 System -vis-à-vis- Halemane's (3.x±1)/4 System

	1	5	7	11	13	17	19	23	25	29	31	35	37	41	43	47	49	53	
19	174763 (0,1)	873813 (0,3)	1223339	1922389	2271915	2970965	3320491	4019541	4369067	5068117	5417643	6116693	6466219	7165269	7514795	8213845	8563371	9262421	19
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17	43691 (0,5)	218453 (0,5)	305835	480597	567979	742741	830123	1004885	1092267	1267029	1354411	1529173	1616555	1791317	1878699	2053461	2140843	2315605	17
16	21845 (1,5)	109227 (0,3)	152917	240299	283989	371371	415061	502443	546133	633515	677205	764587	808277	895659	939349	1026731	1070421	1157803	16
15	10923 (2,3)	54613 (0,1)	76459	120149	141995	185685	207531	251221	273067	316757	338603	382293	404139	447829	469675	513365	535211	578901	15
14	5461 (1,1)	27307 (0,1)	38229	60075	70997	92843	103765	125611	136533	158379	169301	191147	202069	223915	234837	256683	267605	289451	14
13	2731 (2,1)	13653 (1,3)	19115	30037	35499	46421	51883	62805	68267	79189	84651	95573	101035	111957	117419	128341	133803	144725	13
12	1365 (2,3)	6827 (0,5)	9557	15019	17749	23211	25941	31403	34133	39595	42325	47787	50517	55979	58709	64171	66901	72363	12
11	683 (2,5)	3413 (1,5)	4779	7509	8875	11605	12971	15701	17067	19797	21163	23893	25259	27989	29355	32085	33451	36181	11
10	341 (2,5)	1707 (2,3)	2389	3755	4437	5803	6485	7851	8533	9899	10581	11947	12629	13995	14677	16043	16725	18091	10
9	171 (1,3)	853 (1,1)	1195	1877	2219	2901	3243	3925	4267	4949	5291	5973	6315	6997	7339	8021	8363	9045	9
8	85 (2,1)	427 (2,1)	597	939	1109	1451	1621	1963	2133	2475	2645	2987	3157	3499	3669	4011	4181	4523	8
7	43 (1,1)	213 (2,3)	299	469	555	725	811	981	1067	1237	1323	1493	1579	1749	1835	2005	2091	2261	7
6	21 (0,3)	107 (2,5)	149	235	277	363	405	491	533	619	661	747	789	875	917	1003	1045	1131	6
5	11 (1,5)	53 (2,5)	75	117	139	181	203	245	267	309	331	373	395	437	459	501	523	565	5
4	5 (0,5)	27 (1,3)	37	59	69	91	101	123	133	155	165	187	197	219	229	251	261	283	4
3	3 (0,3)	13 (2,1)	19	29	35	45	51	61	67	77	83	93	99	109	115	125	131	141	3
2	1 (0,1)	7 (1,1)	9 (1,3)	15 (2,3)	17 (2,5)	23 (0,5)	25 (1,1)	31 (2,1)	33 (2,3)	39 (0,3)	41 (0,5)	47 (1,5)	49 (2,1)	55 (0,1)	57 (0,3)	63 (1,3)	65 (1,5)	71 (2,5)	2
1		3		7		11		15		19		23		27		31		35	1
	1	5	7	11	13	17	19	23	25	29	31	35	37	41	43	47	49	53	

Halemane's (3.x+1)/4 System - as a clone of the Collatz System

Start with any given positive integer x>1; if (x=1) STOP;
elseIf x≡0(mod4) then, divide by 4 repeatedly until the resultant x≢0(mod4);
ElseIf x≡2(mod4) then, multiply by 2 to get a resultant x≡0(mod4);
ElseIf {x≡1(mod4)}∨{x≡3(mod4)} then, multiply by 3 and add 1 to get a resultant {x≡0(mod4)}∨{x≡2(mod4)}; Now continue the process.

Halemane's (3.x+1)/4 Function

if (x=1) STOP;
elseIf x≡0(mod4) then, S(x) ≔ (x/4); elseIf x≡2(mod4) then, S(x) ≔ (2x);
elseIf {x≡1(mod4)}∨{x≡3(mod4)} then, S(x) ≔ (3.x+1);

	$[0]_4$	$[1]_4$	$[2]_4$	$[3]_4$
$[0]_4$	0	1	1	1
$[1]_4$	1	0	0	0
$[2]_4$	1	0	0	0
$[3]_4$	0	0	1	0

Halemane's (3.x+1)/4 System - modified

Start with any given positive integer x>1; if (x=1) STOP;
elseIf x≡0(mod4) then, divide by 4 repeatedly until the resultant x≢0(mod4);
ElseIf x≡2(mod4) then, divide by 2 to get a resultant x≢2(mod4);
ElseIf {x≡1(mod4)}∨{x≡3(mod4)} then, multiply by 3 and add 1 to get a resultant {x≡0(mod4)}∨{x≡2(mod4)}; Now continue the process.

Halemane's (3.x+1)/4 Function

if (x=1) STOP;
elseIf x≡0(mod4) then, S(x) ≔ (x/4); elseIf x≡2(mod4) then, S(x) ≔ (x/2);
elseIf {x≡1(mod4)}∨{x≡3(mod4)} then, S(x) ≔ (3.x+1);

	$[0]_4$	$[1]_4$	$[2]_4$	$[3]_4$
$[0]_4$	0	1	1	1
$[1]_4$	1	0	0	0
$[2]_4$	0	1	0	1
$[3]_4$	0	0	1	0

Recommended Reading

[§1]. Wikipedia Page – https://en.wikipedia.org/wiki/Collatz_conjecture

[§2]. Richard K Guy; "Don't Try to Solve These Problems";
The American Mathematical Monthly: Vol. 90, No. 1, pp. 35-41; 1983.
https://doi.org/10.1080/00029890.1983.11971148

[§3]. Jeffrey C Lagarias; "The 3x+1 problem: An annotated bibliography (1963--1999); sorted by author";
https://arxiv.org/abs/math/0309224

[§4]. Jeffrey C Lagarias; "The 3x+1 Problem: An Annotated Bibliography, II (2000-2009)";
https://arxiv.org/abs/math/0608208

[§5]. Jeffrey C Lagarias; "The 3x + 1 Problem: An Overview";
https://arxiv.org/abs/2111.02635

[§6]. Jeffrey C Lagarias; "The Ultimate Challenge: The 3x+1 Problem";
American Mathematical Society; 2010.

[§7]. Keshava Prasad Halemane; "Collatz-Conjecture Proved and Halemane-Conjecture Proposed";
https://doi.org/10.31224/6063; 2025.

[§8]. Keshava Prasad Halemane; "Halemane's Theory on the Collatz (3x+1) System";
https://doi.org/10.31224/7569; 2026.

EXPLORATORY-ROUGH-WORK

[6k-1] = [4k-1](UD);

if $[m](UD)^k = n$; then $(n+1)/(m+1) = (3/2)^k$;
or equivalently, $(n+1).2^k = (m+1).3^k$;
or equivalently, $[2.2^k -1](UD)^k = [2.3^k -1]$;

If $x \equiv 0 \pmod 2$; let $x = (a.2^p -1).2^q$ for some $a \equiv 1 \pmod 2$ and $0<p$ and $0<q$;
then, $x \Leftarrow [x](D)^q = [x]/2^q = (a.2^p -1)$; which is an odd number;
ElseIf $x \equiv 1 \pmod 2$ and if $x = (a.2.2^p -1)$ for some $a \equiv 1 \pmod 2$ and $0 \leq p$;
then, $x \Leftarrow [x](UD)^q = (a.2.3^p -1$; which is an odd number.

[6k+1] = [8k+1](UDD) ;

if $[m](UDD)^k = n$; then $(n-1)/(m-1) = (3/4)^k$;
or equivalently, $(n-1).4^k = (m-1).3^k$;
or equivalently, $[4^k +1](UDD)^k = [3^k +1]$;

If $x \equiv 0 \pmod 2$; let $x = (a.2^p -1).4^q$ for some $a \equiv 1 \pmod 2$ and $0<p$ and $0<q$;
then, $x \Leftarrow [x](DD)^q = [x]/4^q = (a.2^p -1)$; which is an odd number;
ElseIf $x \equiv 1 \pmod 2$ and if $x = (a.4.4^q +1)$ for some $a \equiv 1 \pmod 2$ and $0 \leq q$;
then, $x \Leftarrow [x](UDD)^q = (a.4.3^q -1)$; which is an odd number.

Fruits of the CTUHSK Arborescence

Odd multiples of 3 reached through condensed compact inverse Collatz function applied on BEL(6b+1); 0≤b; (j_3-b_3)=3c;
hanging from the rung defined by the binary-exponent-value k:=(2j):=2(3c+b_3); 0≤i;
and
BEL(6b+5); 0≤b; (j_3+b_3-1)=3c;
hanging from the rung defined by the binary-exponent-value k:=(2j-1):=1+2(3c-b_3); 0≤i;

$$z = (6b + 1) \Leftarrow (8b + 1) = y = (6a - 1) \Leftarrow (4a - 1) = x$$
$$4(z - 1) = 3(y - 1) \text{ and } 2(y + 1) = 3(x + 1)$$
$$8(z - 1) = 6(y - 1) \text{ and } 6(y + 1) = 9(x + 1)$$
$$2(4z - 1) = 6(y) = 3(3x + 1)$$

z	z⇐y	y	!=!	y	y⇐x	x
1	z⇐y(UDD)D^2	5		5	y⇐x(UD)	3
5	z⇐y(UDD)D	13		5	y⇐x$(UD)^4(D)^3$	15
13	z⇐y(UDD)	17		17	y⇐x(UD)	11
7	z⇐y(UDD)	9		11	y⇐x(UD)	7
11	z⇐y(UDD)D	29		29	y⇐x(UD)	19
19	z⇐y(UDD)	25		35	y⇐x(UD)	23
25	z⇐y(UDD)	33		47	y⇐x(UD)	31
31	z⇐y(UDD)	41		41	y⇐x(UD)	27
z	z⇐y	y		y	y⇐x	x

1⇐5$(UDD)^2$; 5⇐13(UDD)D; 13⇐17(UDD); 17⇐11(UD); 11⇐7(UD); 7⇐9(UDD);
5⇐3(UD); 5⇐15$(UD)^4(D)^3$; 1⇐21(UDD)$(D)^4$;